A new ensemble empirical mode decomposition (EEMD) denoising method for seismic signals

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Abstract

This paper suggests a new denoising technique based on the Ensemble Empirical mode decomposition (EEMD). This technique has been compared with the discrete wavelet transform (DWT) thresholding. Firstly, both methods have been implemented on synthetic signals with diverse waveforms ('blocks', 'heavy sine', 'Doppler', and 'mishmash'). Secondly, the denoising methods have been applied on real seismic traces recorded in the Algerian Sahara. It is shown that the proposed technique outperforms the DWT thresholding. In conclusion, the EEMD technique can provide a powerful tool for denoising seismic signals.

1. Introduction

Denoising is a critical step in seismic signal processing. Many techniques have been suggested to this purpose: highlighting the wanted information and attenuating the undesired signal (noise).

Time-frequency analysis techniques, such as Short Time Fourier Transform (STFT) or Wavelets are revealed to be appropriate to handle non-stationary signals. They can be performed in time and frequency domain. Applications

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show that both the optimizing selection of wavelet basis, and the decomposition level (for discret wavelet transform, DWT) may cause large inconvenience to the application of wavelet denoising.

Lately, Empirical Mode Decomposition (EMD) has been suggested by N. Huang [9]. It allows to decompose the non-linear and non-stationary signal into multiple intrinsic mode functions (IMF), without requiring a priori basis function. It has been successfully applied for signal denoising in various research fileds: biomedical signals [19], acoustic signals [13], and ionospheric signals [18]. However, the effectiveness of the EMD method is affected by the mode mixing effect (modal aliasing) [11]. To overcome this drawback, Wu and Huang [21] proposed a noise-assisted EMD algorithm, named ensemble empirical mode decomposition (EEMD).

This research suggests a EEMD-based method for Gaussian noise removal. This method has been compared with DWT thresholding denoising technique via applications on simulated and real datasets. The rest of this paper is structured as followed. A brief theory on the denoising methods is first given. Then, section 3 and 4 present the results obtained by these techniques from synthetic data and real seismograms recorded in the Algerian Sahara, respectively. Finally, the concluding remarks are given in section 5.

2. Theory

2.1. EMD algorithm

Any non-linear and non-stationary data set can be adaptively decomposed into IMF components via EMD method, without the need to a priori basis as are Fourier and wavelet-based methods [4] [9] [10]. The term IMF, is the monocomponent function or an oscillatory mode with one instantaneous frequency, that needs to satisfy two criteria:

a) In the whole time series, the number of extrema and the number of zero crossings must be either equal or differ at most by one.

b) At any point in the time series, the mean value of the envelopes which is defined by local maxima (upper envelope) and local minima (lower envelope) is equal to zero.

The process of extracting IMFs via EMD is called sifting algorithm, which is an iterative method detailed as follows:

Step 1: Identify the extrema (local maxima and minima) of the observed signal x (t).

Step 2: Interpolate the local extrema using cubic spline to obtain the upper U(t) and the lower L(t) envelopes.

Step 3: Calculate the local mean value of the upper and the lower envelopes $m(t) = (U(t) + L(t))/2$

Step 4: Subtract the local mean m(t) from the from the original signal: $h_1(t) = X(t) - m(t)$

Step 5: Replace the signal x(t) by h_1(t), and reiterate Steps 1–4 until the obtained signal satisfies the two IMF conditions (a) and (b) mentioned above.

One of these criteria is used to stop the sifting process: after extracting (M-1) IMFs, the residue, $r_M(z)$ is either an IMF or a monotonic function . Stopping criteria are discussed by previous researches [4] [9] [10] [11] [12] [15] [16].

The original signal can be then reconstructed by superposing the obtained IMFs:

$$X(t) = \sum_{m=1}^{M-1} \text{IMF}_m(t) + r_M(t) \tag{1}$$

where M-1 is the number of IMFs, i.e. the signal is decomposed into (M-1) IMFs and one residual.

2.2. EEMD algorithm

As aforementioned, the major shortcoming of EMD is the effect of mode mixing. This phenomenon occurs when the oscillations with disparate time scales are preserved in one IMF, or the oscillations with the same time scale are sifted into different IMFs.

To overcome the mode mixing obstacle, Wu and Huang [21] suggested a noise-assisted EMD algorithm, named EEMD, which allows a better scale separation aptitude than the standard EMD method. The EEMD consists of adding different series of white noise into the signal in several trials. Since the added noise is different in each trial, the resulting IMFs don't exhibit any correlation with the corresponding IMFs from one trial to another. If the number of trials is adequate, the added noise can be eliminated by ensemble averaging of the obtained IMFs related to the different trials.

The specific steps of EEMD algorithm are as follows [21]:

Step i : In the nth trial, a new time series is generated by adding a white noise time series $u_n(t)$ to a given signal $x(t)$
$Y_n(t) = X(t) + u_n(t)$, for n = 1, 2, . . . ,N, with N the ensemble number.

Step ii: Based on the original EMD, the noise-contaminated signal $Y_n(t)$ is decomposed into a set of IMFs and a residual

$$Y_n(t) = \sum_{m=1}^{M-1} \text{IMF}_m^{(n)}(t) + r_M^{(n)}(t) \tag{2}$$

where M-1 is the total number of the IMFs resulting in each decomposition of $Y_n(t)$, $\text{IMF}_m^{(n)}$ is the mth IMF and $r_M^{(n)}$ is the residual obtained in the nth trial. In order to an equal number of IMFs in each decomposition, a fixed sifting number of 10 is considered.

Step iii. The steps (i) and (ii) are reiterated for N trials. In each trial, a different white noise series $u_n(t)$ is added to the original signal.

Step iv. The final IMF of the EEMD (IMF_m^{ave}) is obtained by averaging the total m IMF related to N trials:

$$\text{IMF}_m^{ave}(t) = \frac{1}{N} \sum_{n=1}^{N} \text{IMF}_m^{(n)}(t) \tag{3}$$

The results achieved by the EEMD depend on the choice of the ensemble number (N) and the amplitude of added noise (A). It is shown that the following relation should be satisfied [21]:

$$\varepsilon = \frac{A}{\sqrt{N}}$$

with ε being the final standard deviation of error calculated as the difference between the original signal and the sum of the IMFs resulting from the EEMD.

In the following, the standard deviation of the added noise series equals 0.2 times the standard deviation of the raw data, while The ensemble number was set to N = 100.

2.3. EMD and EEMD denoising methods based on a Gaussian noise model

Here, the suggested denoising method is based on a threshold selection of IMFs, and the threshold is determined depending on the energy of each mode of the noise. It is stated that in the presence of an additive white Gaussian noise (noise-only model), the logarithm-variance of each IMF evolves linearly with a parameter, called the Hurst exponent H ([5] [20]),

$$\log_2 V_H(i) = \log_2 V_H(2) + 2(H-1)(i-2)\log_2 \rho_H \tag{4}$$

For $i \geq 2$ and ρ_H =2.01 with $V_H(i)$ is the variance of the ith IMF. Then the IMFs energy can be expressed by:

$$E_k = \left(E_1 \, 2.01^{-k}\right)/0.719, \qquad k = 2,3,...,n-1 \tag{5}$$

where E_1 is estimated by the variance of the first IMF of the noisy signal,

Using the above model, Flandrin et al. [6] propose an EMD-based denoising scheme. The latter consists of implementing EMD to separate the noisy data into IMFs. Then, the energies of the obtained IMFs are calculated and compared with the theoretical noise-only IMF energies (equation 5). The final denoised data are obtained by adding up the IMFs whose energy exceeds the theoretical noise IMF energy.

Here, a new EEMD-based denoising approach is suggested to differentiate between IMFs corresponding to the noisy data and the noise-only signal. A different threshold is set for the difference between logarithmic values of noisy signal IMFs energies ($E(IMF_k)$)) and   noise-only IMF energies (E_k). When the difference is smaller than the threshold, the IMF is processed as a noise component and discarded in signal reconstruction. Otherwise, it is retained. In the following, the threshold is set to $|0.01 \log_2 E_1|$.

2.4. Discrete wavelet transform

In wavelet transform, the analyzed signal is written as linear combinations of the product of the wavelet coefficients and a mother (or analyzing) wavelet ψ. The is a function having a zero mean and finite energy ([1] [7]). The wavelet family ψ_{ab} is created by translations and dilatations of the wavelet ψ:

$$\psi_{ab}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \quad a > 0, \ b \in R \tag{6}$$

The parameters ''a'' and ''b'' are the scale and the location , respectively. One way to discretize them is:

$$a_m = a_0^{\ m}, \ b_n = nb_0 a_0^{\ m} \ ; \ m,n \in Z \tag{7}$$

with $a_0 > 1$ being a fixed dilation coefficient and $b_0 > 0$ chosen with respect to the mother wavelet ψ.   The particular discretization selection: $a_0 = 2$ and $b_0 = 1$ results in orthogonal bases (given by Eq. 6) :

$$\psi_{mn}(t) = a_0^{-m/2} \ \psi(a_0^{-m} t - nb_0) \tag{8}$$

Using such orthonormal wavelet basis, a function $f \in L^2(R)$ can be expressed by linear combinations [14]:

$$f(t) = \sum_{m \in Z} \sum_{n \in Z} C_{mn} \psi_{mn}(t) \tag{9}$$

with $C_{mn} = \displaystyle\int_{-\infty}^{+\infty} f(t).\overline{\psi}_{mn}(t).dt$ \hfill (10)

where the overlain symbol "-" indicates complex conjugate.
From a mathematical point of view, the DWT can be rewritten as a matrix multiplication: expressed as the product of the discrete samples of the signal and the orthonormal matrix W:
$$c = W.y \tag{11}$$
where W is the orthonormal matrix, c and y are the discrete samples of the signal and the DWT, respectively.

The wavelet thresholding is carried out as follows: $\widetilde{x} = W^{-1}\left(T(Wy)\right)$
1- Computing the DWT coefficients of the signal (W.y),
2- Thresholding the DWT coefficients using the the thresholding operator (T(.)),
3- Reconstructing the denoised signal ($\widetilde{x}$) using the inverse wavelet transform (W^{-1}) .

There are 2 popular types of   the thresholding operator T(·): the hard-thresholding function (T_λ^h) and the soft-thresholding function (T_λ^s).

$$T_\lambda^h = \begin{cases} c, & if \ |c| \geq \lambda \\ 0, otherwise \end{cases} \qquad T_\lambda^s = \begin{cases} sign(c)\left(|c| - \lambda\right), & if \ |c| \geq \lambda \\ 0, & otherwise \end{cases} \tag{12}$$

The threshold λ is a parameter depending on the signal and noise energies. Its selection affects the denoising operation. A small λ value results in a denoised signal very close to the original signal, whereas a large value yield an oversmoothed signal. Four threshold selection rules are considered are considered in this study: ([2] [3] [17]): 'minimaxi', 'universal', 'rigorousSURE' (rigsure), 'heuristicSURE' (heursure).
Regarding threshold rescaling, the multiple scale dependent threshold option is used. It is adaptive and the threshold value is computed for each scale.

3. Application on synthetic data

To evaluate the performance of the denoising process using the suggested methods (the EEMD-based denoising and the DWT thresholding), synthetic data were used. The simulated signals were created using different waveforms: 'blocks', 'heavy sine', 'Doppler', and 'mishmash'. The criterion used to assess the denoising performance is the

mean square error (MSE): $MSE = \dfrac{1}{N} \sum_{i=0}^{N-1} (\tilde{x}_i - x_i)^2$.

A small MSE value indicates that the estimated signal $\tilde{x}$ is close to *x.* Therefore, the smaller MSE value, the more efficient the denoising process. As regards denoising using the wavelet thresholding technique, the optimal parameters (thresholding function and threshold selection rule, wavelet, and level) corresponding to 'blocks', 'heavy sine', and 'Doppler' signals are taken from the results obtained by Honório et al. [8], while those related to 'mishmash' signal were established after many tests.

For each considered signal type, a hundred (100) realizations of gaussian white noise were performed. For each realization, amounts of noise are added to the free-noise signal to get the desired SNR value. Here, the SNR refers to the ratio of the maximum amplitude of the signal to the maximum amplitude of random noise. In this study, seven different SNR values are considered (SNR=15, 10, 5, 3, 2.5, 2, 1.5). For a specified SNR value, the obtained noisy signal is denoised using the techniques discussed above, and the procedure is reiterated for all the 100 realizations. The MSE corresponding to the considered SNR value is obtained by averaging the MSE values derived from the 100 realizations.

Table 1 gives the MSE values calculated using the discussed denoising techniques for the different signals. A worth noteworthy statement is that for 'blocks', 'heavy sine' and 'mishmash' signals, EEMD provides the most efficient denoising results for all the considered SNR values. However, for 'Doppler' signal, the DWT thresholding yields the best results, and the deviation between MSE values obtained by DWT and EEMD is small.

For illustration, the ''blocks'' signal is considered with a SNR=3. The results obtained by the discussed denoising techniques implemented on this signal are shown in Figure 1. As expected, the EEMD denoising method outperforms the wavelet thresholding in term of efficiency.

Table 1. MSE for wavelet thresholding, and EEMD denoising methods calculated for the different input signals

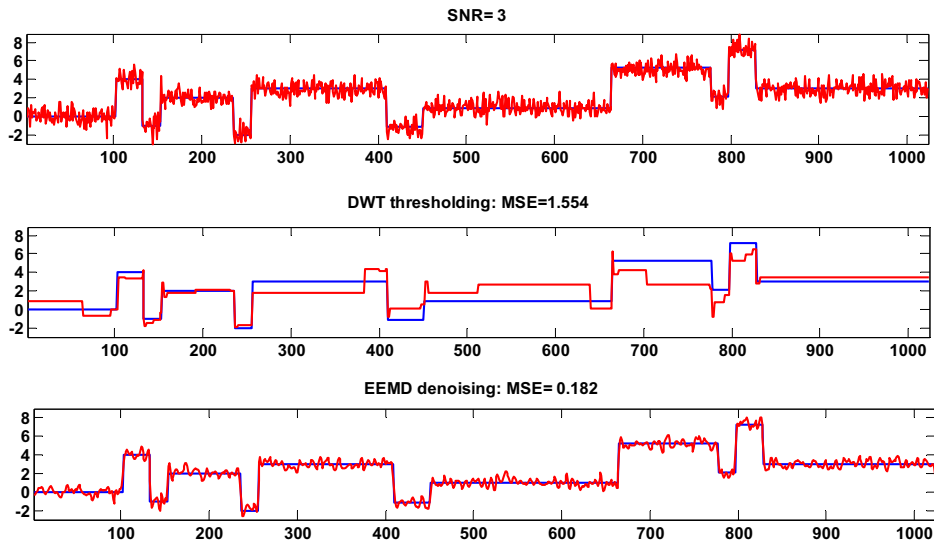| Signal	Method	SNR						
		15	10	5	3	2.5	2	1.5
Blocks	DWT	2.369	2.365	2.533	3.100	3.345	3.529	3.885
	EEMD	0.172	0.184	0.263	0.443	0.557	0.737	1.304
Heavy sine	DWT	0.335	0.339	0.351	0.360	0.378	0.377	0.394
	EEMD	0.012	0.015	0.033	0.057	0.074	0.092	0.154
Doppler	DWT	0.000	0.000	0.001	0.002	0.003	0.003	0.006
	EEMD	0.008	0.008	0.007	0.007	0.007	0.007	0.010
Mishmash	DWT	0.758	0.764	0.778	0.844	0.865	0.964	1.099
	EEMD	0.758	0.757	0.772	0.812	0.856	0.888	0.980

Fig. 1. Results obtained by wavelet thresholding and the suggested EEMD-based denoising technique from noisy Blocks signal SNR= 3. Panel 1: noisy signal (in red) and noise-free signal (in blue), panel 2, 3 represent, respectively, the results obtained by DWT, and EEMD denoising: denoised signal (in red) and noise-free signal (in blue).

4. Application on real seismic data

This section shows the results obtained using the discussed denoising techniques on real dataset acquired during a refraction seismic survey in the Algerian Sahara. The analyzed data corresponds to seismograms recorded by a down-hole array of 8 sensors locating at depths varying from 2.5 to 20m depth with a 2.5-m separation depth interval, and a sampling rate of 4 ms.

Here, the results derived from the available real seismic dataset using the different methods are compared in terms of efficiency. In this view, simulated random noise is created and added to the noise-free raw seismograms to get an SNR value of 3 (Figure 2). The denoised seismograms obtained from the different methods are shown in Figure 3.
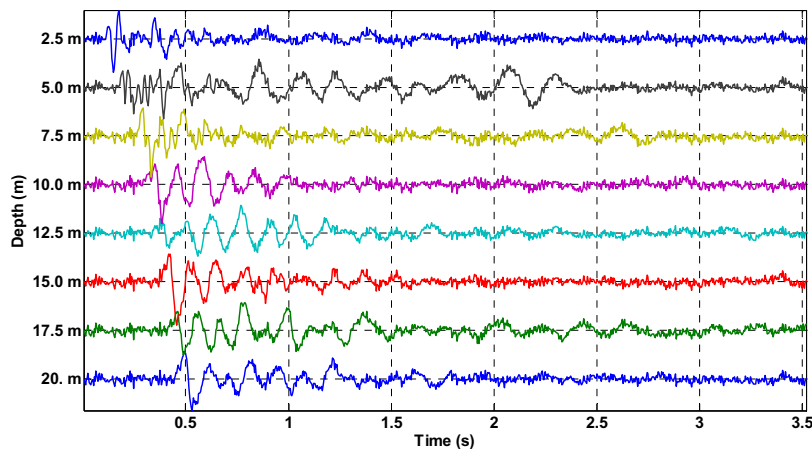

Fig. 2. Real seismograms recorded in the Algerian Sahara.

In order to assess the performance of each method, MSE values are computed for each seismogram. Table 2 shows that the more efficient denoising is obtained by EEMD for the most part of the dataset (seismograms 1,2, 3, 5, 7,8). However, the least MSE values are yielded using DWT for seismograms 4 and 6; these values are slightly different

from those corresponding to EEMD. Overall, EEMD can be considered as the best denoising technique. In contrast with DWT requiring the selection of different parameters (wavelet, thresholding function, threshold selection rule, and threshold rescaling option), EEMD is revealed to be more adaptive method.

Table 2. MSE for wavelet denoising and EEMD methods calculated for the considered seismograms.

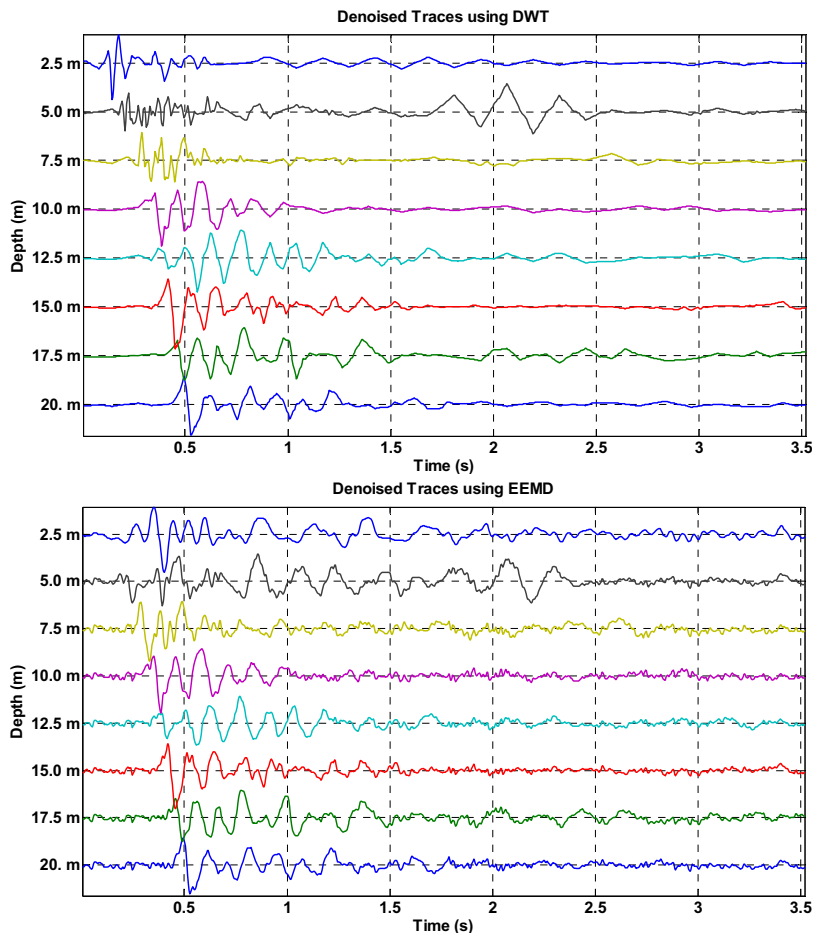| Depth (m)	2.5	5.	7.5	10.	12.5	15.	17.5	20.
Seismogram number	1	2	3	4	5	6	7	8
DWT	0.0016	0.0033	0.0026	0.0012	0.0030	0.0015	0.0054	0.0055
EEMD	0.0009	0.0026	0.0020	0.0016	0.0022	0.0017	0.0030	0.0027

Fig. 3. Denoised seismograms from the real dataset using wavelet thresholding, and EEMD denoising methods.

5. Conclusion

The Denoising is a crucial step in geophysical data processing. This paper proposes a new denoising technique based on the Ensemble Empirical mode decomposition (EEMD).

Here, two denoising techniques (DWT and EEMD) are compared. Applications on synthetic dataset with different waveforms ('blocks', 'heavy sine', 'Doppler', and 'mishmash') show that EEMD is the most efficient for 'blocks', 'heavy sine' and 'mishmash' signals considering different SNR values, while for 'Doppler' signal, the DWT thresholding outperforms the other technique, and the discrepancy between MSE values obtained using the studied methods is very small. Next, applications of the denoising techniques on real dataset demonstrate that EEMD is the most effective for the most number of seismograms. However, DWT outperforms EEMD for seismograms 4 and 6, and the different from MSE values are very small. To conclude, the EEMD-based denoising technique can be effectively applied to enhance the efficacy of seismic signal denoising.

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